Results and discussion: Fig. 1 shows theoretical curves of μ_4', as given by eqn. 11, against β for various values of q_0. It is assumed that $q_2 = 2q_1 = 4q_0$, and other terms in the summation are neglected. For $q_0 = \infty$, it can be seen that μ_4' is relatively small for all β above, say, 40. Thus, for long sequences where large β are used, it may be sufficient to ensure that q_0 is not less than the optimum predicted by eqn. 13.

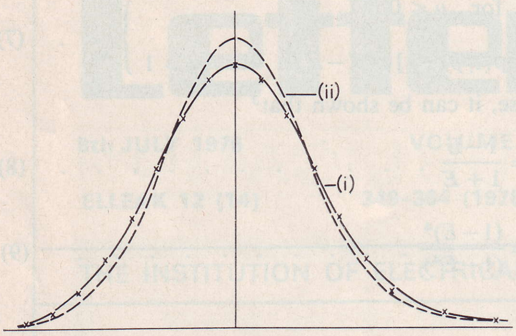

Fig. 2 *Amplitude probability-density distributions of filtered 17-stage sequences*

$\beta = 44$
— — — — octal = 774017, $\beta_0 = 11$
———— octal = 655747, $\beta_0 = 44$
× × × × Gaussian

Table 1 *m*-SEQUENCES CHOSEN TO GIVE CLOSE APPROXIMATION TO GAUSSIAN DISTRIBUTION

β	Register length	Octal	q_0
	13	32563	23
	14	46773	26
10	15	170361	26
	16	313425	28
	17	777603	27
	13	27373	50
	14	75053	49
20	15	124431	51
	16	337521	49
	17	452525	52
40	17	655747	111

β = required ratio of filter time constant to clock period
q_0 = lowest value of the sum $i+j+k$ for the quadrinomials of the form $x^i+x^j+x^k$ divisible by the characteristic polynomial

Thus selection of a sequence to give a distribution which approximates to Gaussian can be achieved as follows. The sequences which exhibit skew are first eliminated.[1,2] The q_0 and q_1 are then determined for the remaining sequences to select the one which most closely satisfies the conditions for μ_4' to be zero. This technique was applied to the sequences in the range of register lengths from 13 to 17 and the results are listed in Table 1, with the characteristic polynomials given in octal notation, as used by Peterson.[5]

The importance of correct choice of characteristic polynomial at a given β is demonstrated in Fig. 2, which shows a comparison of the distributions of two 17-stage sequences subjected to 1st-order filtering with $\beta = 44$. Sequence (i) has $\beta_0 = 44$ and gives a distribution which closely approximates Gaussian. Sequence (ii), however, has $\beta_0 = 11$, and, at $\beta = 44$, the deviation from Gaussian is clearly seen.

G. H. TOMLINSON
P. GALVIN

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Department of Electrical Engineering
University of Salford
Salford M5 4WT, England

References

1 TOMLINSON, G. H., and GALVIN, P.: 'Analysis of skewing in amplitude distributions of filtered *m*-sequences', *Proc. IEE*, 1974, **121**, (12), pp. 1475–1479
2 TOMLINSON, G. H., and GALVIN, P.: 'Elimination of shewing in the amplitude distributions of long *m*-sequences subjected to lowpass filtering', *Electron. Lett.*, 1975, **11**, pp. 77–78
3 GALVIN, P.: 'Some properties of filtered pseudorandom sequences'. Ph.D. thesis, University of Salford, 1974
4 WAINBERG, S., and WOLF, J. K.: 'Subsequences of pseudorandom sequences', *IEEE Trans.*, 1970, **COM–18**, pp. 606–612
5 PETERSON, W. W.: 'Error-correcting codes' (Wiley, 1961), Appendix C

COMPUTERISED SEARCH FOR LINEAR BINARY CODES

Indexing terms: Codes, Communications applications of computers

A computerised search procedure is described for finding new binary codes. The method involves the extension of a given known code by annexing a number of parity-check digits to it in such a way that the minimum Hamming distance of the given code is improved. A number of codes found by this procedure have better rates than the best known codes of identical Hamming distance and the same number of information digits; a table of these codes is presented.

Introduction: Many techniques of code modification have been reported in the literature;[1,2] one such technique, code extension, involves the annexation of extra parity checks to the (n, k) code; even-Hamming-distance codes may be constructed in this way by annexing an overall parity-check digit to the appropriate linear block codes of odd Hamming distances. In general, more than one parity-check digit is needed to increase the Hamming distance by at least 2. In this letter, a computerised search procedure is developed to determine the minimum number of parity-check digits required to increase the distance of a given code by a specified amount.

In the codebook[2,3] of an (n, k) linear binary code, the parity-check digits represent $n-k$ column vectors chosen from a possible $M = 2^k-1$ vectors; the M vectors represent all linear (modulo-2) combinations of the k information columns. One obvious procedure therefore for finding the parity-check digits needed to increase a given code distance by a factor of, say, m is to test every combination of $\binom{M}{i}$ possible parity-check digits for $i = m, \ldots, M$, until the required code distance is obtained. The determined i-parity check digits are then the needed minimum check digits to obtain the required Hamming distance. This exhaustive procedure is only feasible for very small values of k, since, for larger values, the computer time needed is excessive. The test may, nevertheless, be done on a small percentage selected at random of the total possible combinations $\binom{M}{i}$ of parity-check digits. Satisfactory results have been obtained in this way. In this letter, a third procedure is described, which has been found to yield a number of new codes within a reasonable amount of computing time.

Computer search: The search involves the following steps: Starting with a given (n, k) codebook of even Hamming distance, say, d,

(*a*) Determine the parity-check digit which increases the weight of a maximum number of codewords of minimum weight. The parity-check digit is selected randomly.

(*b*) Increase the length of the given code by adding the new parity-check digit and form the new column of the codebook.

(*c*) Test whether the new code has a distance $d+1$. If not, go back to step *a*.

(*d*) Print out the basis vectors of the newly found code.

Table 1

New (n, k, d) code	d_l, d_u	Best known (n, k, d)* code	Parity-check columns of $[G]$ matrix in octal
23, 7, 9	8, 9	24, 7, 9	75, 166, 17, 51, 122, 140, 54, 107, 52, 121, 67, 76, 115, 116, 124, 165
31, 7, 13	12, 14	32, 7, 13 31, 6.17, 13	All columns of the above (23, 7, 9) code plus: 33, 130, 53, 150, 63, 177, 113, 155
40, 7, 17	16, 18	40, 6.46, 17 41, 7, 17	All columns of the above (31, 7, 13) code plus: 161, 143, 64, 156, 71, 34, 46, 125, 25
74, 7, 35†	34, 36	75, 7, 35	125, 27, 73, 144, 76, 167, 17, 31, 142, 52, 14, 32, 172, 41, 63, 75, 116, 101, 173, 104, 146, 112, 130, 117, 122, 150, 74, 3, 127, 154, 6, 131, 7, 24, 160, 21, 15, 36, 44, 135, 45, 165, 134, 57, 56, 155, 126, 66, 143, 71, 176, 113, 123, 11, 33, 25, 70, 62, 161, 67, 53, 37, 105, 164, 151, 177, 147
78, 7, 37†	36, 38	81, 7, 37	All columns of the above (74, 7, 35) code plus: 103, 22, 174, 65
82, 7, 39†	38, 40	83, 7, 39	All columns of the above (78, 7, 37) code plus: 50, 141, 60, 171
86, 7, 41‡	40, 42	88, 7, 41	167, 21, 6, 140, 123, 65, 104, 53, 74, 115, 176, 17, 30, 132, 153, 13, 23, 117, 136, 34, 46, 15, 57, 163, 111, 142, 155, 106, 144, 130, 32, 61, 172, 127, 165, 51, 70, 121, 67, 76, 105, 122, 54, 171, 114, 135, 47, 177, 24, 12, 37, 3, 7, 26, 31, 35, 43, 45, 56, 60, 62, 64, 71, 73, 75, 101, 110, 112, 124, 133, 137, 141, 145, 150, 154, 156, 160, 166, 173
89, 7, 43‡	42, 44	90, 7, 43	All columns of the above (86, 7, 41) code plus: 16, 42, 103
42, 8, 17	16, 18	43, 8, 17	327, 150, 311, 47, 362, 134, 121, 255, 223, 157, 64, 214, 122, 352, 55, 330, 116, 273, 5, 51, 32, 212, 274, 224, 13, 347, 170, 177, 371, 62, 264, 256, 317, 344
75, 8, 33	32, 36	77, 8, 33	345, 166, 260, 236, 111, 171, 265, 106, 325, 46, 212, 103, 117, 206, 310, 203, 316, 205, 165, 51, 76, 142, 272, 177, 304, 13, 54, 156, 45, 155, 224, 217, 362, 52, 62, 323, 227, 354, 214, 274, 332, 114, 211, 376, 242, 16, 57, 172, 144, 25, 255, 307, 31, 235, 357, 320, 135, 32, 373, 221, 67, 70, 133, 364, 136, 375
90, 8, 41	40, 44	93, 8, 41	All columns of the above (75, 8, 33) code plus: 370, 247, 351, 153, 352, 150, 361, 163, 127, 277, 160, 230, 121, 271, 43
94, 8, 43	42, 45	95, 8, 43	All columns of the above (90, 8, 41) code plus: 147, 334, 174, 315
105, 8, 49	48, 52	108, 8, 49	All columns of the above (94, 8, 43) code plus: 26, 130, 302, 331, 73, 112, 343, 263, 346, 250, 222
109, 8, 51	50, 53	112, 8, 51	All columns of the above (105, 8, 49) code plus: 313, 301, 241, 266
113, 8, 53	52, 56	116, 8, 53	All columns of the above (109, 8, 51) code plus: 367, 233, 124, 326
116, 8, 55	53, 56	118, 8, 55	All columns of the above (113, 8, 53) code plus: 23, 122, 61
120, 8, 57	56, 59	121, 8, 57	All columns of the above (116, 8, 55) code plus: 64, 15, 75, 105

(n, k, d) = the newly found code of length n, k information digits and of minimum Hamming distance
d_l, d_u = Helgert and Stinaff lower and upper bounds on minimum Hamming distance
* = best known (n, k, d) code from the recent table of MacWilliams and Sloane[5]
†A linear code of the same n, k and d is found independently by Farrell in unpublished work
‡A linear code of the same n, k and d is reported by Farrell and Farrag[6]

Table 1 contains a list of 17 new linear block codes of odd Hamming distance obtained in this way; another 17 new codes of even Hamming distance, not listed, may be obtained by adding an overall parity-check digit to each of the listed codes. All codes of Table 1 are better than the best existing codes as listed in the table of Helgert and Stinaff[4] and the recent table of MacWilliams and Sloane.[5]

Comments: The new codes given in Table 1 are for $k = 7$ and 8 only. The proposed computer search may, however, be carried out for $k = 9, \ldots, 15$ if a reasonable amount of computer memory and time are available. Nevertheless, it is thought that $k = 15$ presents a practical limit to the proposed computer search.

A. A. HASHIM

Department of Electronic & Communications Engineering
Polytechnic of North London
Holloway, London N7 8DB, England

7th June 1976

V. S. POZDNIAKOV

Department of Electrical Engineering
Imperial College of Science & Technology
London SW7 2BT, England

References

1 BERLEKAMP, E. R.: 'Algebraic coding theory' (McGraw–Hill, New York, 1968)
2 PETERSON, W. W., and WELDON, E. J., JUN.: 'Error-correcting codes' (MIT Press, 1972, 2nd edn.)
3 FARRELL, P. G.: 'Linear binary anticodes', *Electron. Lett.*, 1970, **6**, pp. 419–421
4 HELGERT, H. J., and STINAFF, R. D.: 'Minimum-distance bounds for binary linear codes', *IEEE Trans.*, 1973, **IT-19**, pp. 344–356
5 MACWILLIAMS, F. J., and SLOANE, N. J. A.: 'Coding theory' (North–Holland, to be published)
6 FARRELL, P. G., and FARRAG, A.: 'Further properties of linear binary anticodes', *Electron. Lett.*, 1974, **10**, pp. 340–341

NETWORK-ANALYSER REFLECTION MEASUREMENTS OF MICROSTRIP CIRCUITS NOT REQUIRING EXACTLY REPRODUCIBLE COAXIAL-TO-MICROSTRIP TRANSITIONS

Indexing terms: Microwave measurements, Network analysers, Striplines

A measuring and correction procedure for network-analyser measurements of microstrip circuits is presented which does not need exactly reproducible coaxial-to-microstrip transitions. The transmission properties of the transitions and the parameters of the microstrip lines are determined by transmission measurements of two homogeneous lines; the input reflection coefficient of the coaxial-to-microstrip transition is measured directly at the microstrip circuit under test using sliding-load techniques. Assuming the transition to have low losses, its backward reflection coefficient can be calculated approximately. The measured scattering parameters of the microstrip circuit are corrected using these values, and a measurement example shows the results of this procedure.

Recently, many calibration and correction methods for microwave network-analyser measurements have been described in the literature.[1–6] Several calibration standards are used to determine the parameters of an error network with which the measured reflection and transmission parameters are corrected. When performing measurements on microstrip circuits by these procedures, the coaxial-to-microstrip transitions have an important influence on the measurements and must be reproducible in absolute value as well as in phase of their scattering parameters. These assumptions often are no longer valid with sufficient accuracy